Dynamics of Female Labor Force Participation and Welfare with Multiple Social Reference Groups

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Abstract

I develop a model with status concerns to analyze how different economic factors affect female labor participation and welfare, as well as average household incomes and wages. Reductions in the price of domestic goods and increases in female wages have positive effects on female participation. Increases in male wages have different effects on female participation depending on whether they affect female wages or not. Events that lead to increases in female participation are usually associated with decreases in the welfare of stay-at-home wives but are not necessarily associated with increases in welfare of working wives. Allowing for part-time work can lead to an increase in overall female labor force participation, but some women that would have worked full-time end up working part-time. If female wages are endogenous, an increase in male wages leads to an increase in the female participation rate even if it is not associated with a decrease in the gender wage gap. The positive feedback of increased female participation on their wages can lead to hysteresis of dual equilibria of high and low female labor force participation and a discontinuous transition between these equilibria.

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1. Introduction

Women's labor force participation increased dramatically over the second half of the 20th century. The economic literature has put forward various explanations for this phenomenon, explanations that are mostly associated with developments that by objective measures lead to increases in female welfare (Blau 1998). Recent literature on female labor force participation has focused on cultural changes as an engine of increased participation (Fernandez 2013, Escriche 2007, Vendrik 2003, Fortin 2008). As more women participate in the labor force, the expectations regarding women's role in society adjust and it becomes easier for the subsequent generations of women to join the labor force. By changing the culture that kept married women at home, women were given a wider set of options, allowing them to choose the lifestyle that maximizes their welfare. Surprisingly then, Stevenson and Wolfers (2009) found that women's happiness has declined both absolutely and relative to men during this time. I argue that the "paradox of declining female happiness" can be explained by status concerns with respect to multiple reference groups, in a similar fashion to how "the Easterlin paradox" was justified by relative income concerns (Clark, Frijters and Shields 2008).

To this end, I incorporate status concerns at household and individual level in a model where female labor force participation is affected by gender wage gap and the price of "home goods", external factors that were shown to have contributed to the increase in labor participation of married women throughout the second half of the 20th century[1]. My focus therefore is on how these factors affect married women's decision to join the labor force, the average wage and household income in the economy, as well as the welfare of both stay-at-home and working wives. The results are consistent with the studies that found that decreases in the price of "home goods" relative to female wages and increase in women's wages (decreases in gender wage gaps)

[1] Attanasio, Low and Sanchez-Marcos (2008) focus on labor participation of women belonging to different cohorts during the 20th century and show that decreases in child care costs combined with increases in female wages play an important role in explaining the increase in participation rates. Ferrrero, Martinez and Iza (2004) argue that recent skill-biased technological change led to an increase in skill premium and a relative decrease in the market value of child caring, which they credit with leading to increased participation rates for women. Jones, Manuelli and McGratan (2015) analyze the effects that the decrease in the wage-gender gap, as well as technological improvement in the production of non-market goods had on average hours worked by women of and conclude that reduction in wage-gender gap is the most compelling explanation for the observed increase in female participation. Cardia and Gomme (2018) study the implications of a calibrated version of a life cycle model. They find that increases in the relative wages received by females explain most of the observed changes in labor supply, especially before 1980.

led to increased female participation. Due to the introduction of status concerns, the effects on women's welfare however differ from those typically found in this literature.

The model allows for heterogeneity in women's preference for home production and women's concerns for social status both at household and individual level, and thus potential feelings of envy due to wage gaps between women in the labor force and the men with whom they work. Women who work experience increased welfare because of the income that they receive. They are also subject to decreases in welfare, however, because they must outsource their home production and compete with the men that are already in the workforce[2].

Reductions in the price of "home goods" and increases in female wages have positive effects on the female participation rate. Increases in male wages have different effects on female participation depending on whether male wages affect female wages. If female wage is an independent variable, an increase in male wages decreases the female participation rate. Alternatively, I assume that female wage is a positive function of male wage and past female participation rates[3]. The results change such that an increase in male wages leads to an increase in the female participation rate even if it is not associated with a decrease in the gender wage gap. The positive feedback of increased female participation on their wages can lead to hysteresis of dual equilibria of high and low female labor force participation and a discontinuous transition between these equilibria. The dynamic portion of the model and the hysteresis associated with it is especially relevant in the context of women entering different occupations at different rates based on historical employment patterns.

Most economic models predict that an increase in female labor supply due to positive events in women's lives, such as increased female wages and decreased price of consumption goods, would be associated with increased welfare for working women. Incorporating status concerns at both individual and household level, I find that these events not only are associated with decreases in the welfare of stay-at-home wives but also do not necessarily lead to increases in the welfare of

[2] Women that work might also experience decrease in welfare due to the double burden of work and household chores, but the model allows to combine the two roles in a way that does not increase the amount of work. I assume that women are either working outside the home and buying the home goods from the market or staying at home, and working to produce the home goods themselves. In Section 3.3. I allow for women to combine the two roles by working outside the home on a part-time basis and spending the rest of the working hours at home producing the home goods. There is no labor-leisure choice in the model, as women provide a fixed amount of labor either at home or outside the home.

[3] I make these assumptions because technological advances in the marketplace affect the demand for labor in general, and wages are influenced by the historical trajectory of different categories of workers.

working wives. Additionally, I show that status concerns at the household level, not at the individual level, are the main factor that can lead to a decrease in the happiness of the average working woman.

If part-time work is available to women, overall female labor force participation increases, but some women that would have worked full-time end up working part-time. The effect on the average household income is ambiguous, and therefore the welfare effect on working and stay-at-home women is ambiguous as well.

In the next section I review the related literature on status concerns with respect to different reference groups and the gender wage gap as a function of female participation. In Section 3, I present the model using both exogenous and endogenous female wages, and I develop my propositions. Section 3 also includes a description of possible multiple equilibria and hysteresis effects. Section 4 concludes the paper.

2. Related literature

2.1 Status concerns in relation to different reference groups

The economic literature confirming the importance of income comparisons in the utility function is quite extensive and keeps growing. Using individual level panel data, Luttmer (2005) and Ferrer-i-Carbonell (2005) show that individuals are happier the larger their income is in comparison with the income of their reference group and that higher earnings of neighbors are associated with lower levels of self-reported happiness. Dynan and Ravina (2007) find that people's happiness is positively related to how well they are doing relative to the average in their geographic area. Frank (2007) provides a plethora of anecdotal evidence regarding the effects of status concerns on individual behavior.

The implications of the "keeping up with the Joneses" models are generally used in a gender-neutral fashion to explain saving, consumption, and labor supply decisions at intensive margins at the individual level[4]. Particularly relevant to my research, some authors use these relative income concerns to explain the labor force participation decisions of married women. Neumark and

[4] Abel (1990) and Gali (1994) are concerned with the effects of preference interdependence on asset pricing, Caroll, Overland and Weil (2000), Liu and Turnovsky (2005) are concerned with their effects on capital accumulation, savings and growth, Alvarez-Cuadrado and Long (2012), Garcia-Penalosa and Turnovsky (2008) with their effects on income inequality. Cooper et al (2001) develop a growth model in which agents can consume both "normal goods" that confer direct utility and "status goods" that confer utility at the expense of others. They show that as the economy grows, more resources are transferred to production of status goods which leads to negative utility growth.

Postlewaite (1998) develop a neoclassical model that incorporates relative income concerns into households' utility functions to explain women's employment decisions. They show that these decisions are affected by the incomes of other women with whom relative income comparisons might be important, such as sisters and sisters-in-law (e.g., married women are 16 to 25 percent more likely to have outside employment if their sister's husbands earn more than their own husbands). Park (2010) suggests that married women's employment decisions are influenced by their household's relative income. He shows that married women are more likely to be in the labor force when their husbands' relative income is low, and, in particular, married women who worked previously are more likely to stay in the labor market when their husband's relative income is low. Using data on Italian households, Del Boca and Pasqua (2003) show that the pattern of female employment during 1977–1998 had the effect of reducing inequality in family incomes.

These studies show that concerns about relative income at the household level have an important impact on women's decision to join the labor force. Their focus is the household income advantage that stay-at-home wives capture when they join the labor force. Working women, however, face an additional reference group once they are in the labor market. As workers, women care about their status within their peers' group. Since these comparisons affect women's job satisfaction and welfare, it is a reasonable assumption to make that they also affect the women's decision to work outside the home or not.

The economic literature identifies two potential outcomes of labor market comparisons for women that might earn a lower wage than their male peers. On the one hand, relatively low wages might make people feel worse off because they feel that their work is not compensated at a "fair" value. On the other hand, relatively low wages might make workers feel better off because they interpret it as a signal of what they could achieve in their profession or work organization. Using data on British workers, Clark and Oswald (1996) show that workers' reported satisfaction levels are weakly correlated with their absolute wages but are significantly negatively correlated with their comparison wage rates. Bygren (2004) focuses on what constitutes the reference wage that affects the job satisfaction. He finds that Swedish workers primarily compare their pay with that of similar others (e.g., others with the same education and work experience) in their occupation and in the labor market as a whole. More importantly for this paper, his empirical analysis shows that the higher the average wage of the reference groups, the lower the probability of being satisfied with one's own wage. Sloane and Williams (2000) and Lévy-Garboua and Montmarquette (2004) find that job satisfaction is negatively related to comparison wages, using data on British and Canadian workers, respectively. Brown, Gardner, Oswald and Qian (2008) show that job satisfaction and well-being depend on relative pay and the ordinal rank of an individual's wage within a comparison group.

Clark, Kristensen and Westergaard-Nielsen (2009) provide evidence that in some cases workers do not object if their peers earn more because they feel that those higher earnings are a good indication of their own future prospects. Using Danish data, they find that job satisfaction is positively correlated both with an individual's own earnings and the average earnings of other workers from within the same organization. However, if the reference group is uninformative with respect to one's earning potential, low relative income is likely to have a negative effect on one's well-being.

2.2 Gender wage gap as a function of female participation

I argue that female participation in the labor market can be associated with positive externalities regarding female wages (i.e., increased female participation increases the return to working of all women). In this section I review two lines of argument that explain why this might happen.

One argument focuses on network theories in the labor market. Several studies (Calvo-Armengol and Jackson 2004, 2007; Buhai and Van der Leij 2008) have addressed the issue of the effects of social networks on employment, wage inequality and occupational segregation. They argue that when individuals have "an inbreeding bias," a tendency to associate more closely with individuals that share the same characteristics, there is sustained inequality in wages and employment rates across different groups. Calvo-Armengol and Jackson (2004) show that two groups (e.g., male and female) with two different networks have different employment rates due to the endogenous decision to drop out of the labor force. Arrow and Borzekowski (2004) show that differences in the network connections between blacks and whites can explain about 15 percent of the unexplained variation in wages.

Given the evidence that differences in social networks have an important effect on wage inequality across groups, the question remains as to whether males and females belong to different labor networks. Empirical evidence shows that even though males and females share what is considered the same social space (i.e., they live in the same neighborhoods and go to the same schools), they do not share their networks in terms of the labor market. Berger (1995) provides evidence that women are at a disadvantage in the labor market due to their inferior social network. She shows that women used mostly female contacts in the labor market in 1982: 30 percent of the women in her study used female contacts and only 17 percent of the women used male contacts. As a comparison, 47 percent of the men used male contacts while only 9 percent used female contacts. In general, she shows that women have lower network intensity: 56 percent of the men used labor contacts and only 47 percent of the women used contacts. Fernandez and Sosa (2005) use a dataset documenting the recruitment and hiring process for an entry-level job at a call center of a large U.S. bank. In an environment that was mostly female-dominated, they found that referrers of both genders tend to produce same-sex referrals: females referred females 75.1 percent of the time and males referred females only 56.1 percent of the time.

Another line of argument, based on statistical discrimination theories, supports the conjecture that female wages increase with female labor force participation (Phelps 1972, Lundberg and Startz 1983, Coate and Loury 1993, etc.). Albanesi and Olivetti (2009) suggest that firms' expectation that women spend more time in home production induces them to offer women a lower wage, which consequently leads to women spending less time in the market, validating firms' beliefs. Dolado, García-Peñalosa and De la Rica, (2013) and Escriche (2007) propose models of self-fulfilling expectations in which gender wage gaps are the result of statistical discrimination. If firms believe that women are more likely to quit when they have children, and training workers for highly paid careers is expensive, they will be less likely to offer the same training opportunities to women as they do to men. This in turn leads to the existence of wage gaps which further discourages female participation and dynamically reinforces the mechanism that associates low participation and high gender wage gaps.

3. **The Model**

I consider a model with discrete generations. Population is a continuum and each individual lives for one period. Each household consists of two adults that remain married, and husbands always work in the market and receive a fixed salary. Wives can work at home and produce "home goods" or they can work in the market, in which case they receive a salary and the "home goods" are purchased in the market. I assume that the level of "home goods" is constant, that it is not a choice variable, and that it can be entirely produced outside the home. Household consumption is made up of non-rival, public "home goods" and market goods to which both spouses have equal access. Each generation of women decides on their labor participation at the beginning of their life based on the male wage, the female wage, and the price of "home goods" prevailing at that time.

In Section 3.2 I analyze the labor supply decisions of women when female wages are exogenous, in Section 3.3 I allow for part-time work, and in Section 3.4. I assume that the female wage depends positively on the male wage and on the female participation rate in the previous period.

3.1 Preferences

Women derive utility from consumption of "home goods," market goods and their own relative standing in the society. The "home goods" (e.g., food and child care) can be produced at home or can be purchased in the market if the woman works. Market goods are purchased only with the husband's income if the woman stays at home, or with whatever household income is left after paying for the "home goods" if the woman works. Women care about their household's relative standing in the society and about their individual relative standing in the workforce if they work.

The utility function of women is:

$$u_i = (1-\gamma\mu_i)\alpha_1 u_1\left(C_{h,i}\right) + \alpha_2 u_2\left(C_{m,i}\right) + \alpha_3 u_3\left(\frac{Y_i^{hh}}{\overline{Y^{hh}}}\right) + \alpha_4 u_4\left(\frac{\gamma Y_i^{ind}}{\overline{Y^{ind}}}\right) \tag{1}$$

Where μ_i reflects woman's *i* disutility from not producing her own "home goods", $C_{h,i}$ represents consumption of "home goods", $C_{m,i}$ represents consumption of market goods, Y_i^{hh} represents the household income, $\overline{Y^{hh}}$ represents the average household income, Y_i^{ind} represents the woman's individual income (wage), $\overline{Y^{ind}}$ represents the average individual income, and $\alpha_k, k = \overline{1,4}$ are the weights that reflect the importance of different components of the utility function. Finally, γ is an indicator function such that

$$\gamma = \begin{cases} 1, \text{ if the woman works outside the home} \\ 0, \text{ if the woman stays at home} \end{cases} \tag{2}$$

The parameters μ_i reflect the fact that a woman's preference for "home goods" purchased in the market is heterogeneous. The "home goods" purchased in the market are generic goods; they deviate from the exact specifications that the woman would have achieved if she was producing the goods herself (e.g., market vs. own child care, formula vs. breastfeeding, supermarket muffins vs. homemade muffins, surrogate mothers vs. pregnancy). Some women care about these deviations more than others and thus derive less utility from the consumption of "home goods" if they buy them in the market, as opposed to producing them themselves. If the woman stays at home and personally produces the "home goods" she derives full utility from their consumption[5].

I assume that all "home goods" could be purchased in the market and that consumption of "home goods" is constant, and equal to C_h. The price of "home goods" is equal to p and can change only exogenously (e.g., government subsidizing child care). I let the price of the market goods be the numeraire. I also assume that all working wives receive an equal wage, w_f, and that all husbands receive an equal wage w_m.[6] Thus, the quantity of market goods consumed in a household is $C_m = w_m + \gamma w_f - p\gamma C_h$.

The household's relative standing in the society depends on the household's implicit income $Y^{hh} = w_m + \gamma w_f + (1-\gamma)pC_h$ compared to the average household income, $\overline{Y^{hh}}$.

The women's relative individual standing in the workforce depends on their individual income, Y^{ind}=γw_f compared to the average wage, $\overline{Y^{ind}}$. As stay-at-home wives do not belong to the workforce, their relative standing in the society is not defined by it (i.e., $u_4(0) = 0$).

3.1.2 Women's labor force participation decision

Women join the labor force if the utility from working exceeds that from staying home. Let $\tilde{\mu}$ be the cut-off for which women with $\mu_i \le \tilde{\mu}$ work and women with $\mu_i > \tilde{\mu}$ stay at home.

Assuming that $\mu_i \sim U[0,1]$, where $U[0,1]$ is a uniform distribution between 0 and 1, and taking $\tilde{\mu}$ as given, the average household income is:

$$\overline{Y^{hh}} = w_m + \int_0^{\tilde{\mu}} w_f d\mu + \int_{\tilde{\mu}}^1 pC_h d\mu = w_m + \tilde{\mu}\, w_f + (1-\tilde{\mu})pC_h \qquad (3)$$

All households have the husband's wage equal to w_m, $\tilde{\mu}$ is the fraction of the households that have wives working and receiving a wage w_f, and $1-\tilde{\mu}$ is the fraction of households that have wives staying at home and producing the "home goods" worth: pC_h.

The average individual income is:

[5] Another way of thinking about it is that women derive an intrinsic utility from producing their own home goods (e.g., playing with one's children or cooking for her family). I could model this intrinsic utility, such that instead of working women losing utility from not producing their home good, stay-at-home wives gain utility from producing their own home goods, and the conclusions of the paper will not change.

[6] Suppose that everyone who decides to join the labor force supplies one unit of labor inelastically. Firms use Ricardian technology with constant marginal productivity. The female wage can be justified to be lower than the male wage either by assuming that firms retain as surplus the difference between the marginal product of labor and the female wage or by assuming that women are less productive than men (Section 2.2).

$$\overline{Y^{ind}} = \frac{\int_0^1 w_m d\mu + \int_0^{\tilde{\mu}} w_f d\mu}{1+\tilde{\mu}} = \frac{w_m + \tilde{\mu} w_f}{1+\tilde{\mu}} \quad (4)$$

The labor force is made up of men who all work and receive a wage w_m, and a fraction $\tilde{\mu}$ of the women who receive a wage w_f.

Based on historical evidence and for the purpose of having an interesting discussion regarding female participation, I assume that:

$$w_f > pC_h \quad (5a)$$

$$w_m \geq w_f. \quad (5b)$$

Plugging (3), (4) and the budget constraints into (1), and choosing functional forms for the utility function, the utility function of women can be rewritten as:

$$u_i = (1 - \gamma\mu_i)\alpha_1 C_h + \alpha_2 ln\left(w_m + \gamma w_f - p\gamma C_h\right) + \alpha_3 ln\left(\frac{w_m + \gamma w_f + (1-\gamma)pC_h}{w_m + \tilde{\mu}\, w_f + (1-\tilde{\mu})pC_h}\right) + \alpha_4 ln\left((1 - \gamma) + \gamma \frac{w_f(1+\tilde{\mu})}{w_m + \tilde{\mu} w_f}\right) \quad (1')$$

Given the log utility function, the third term is always positive for working women as $w_f > pC_h$ and thus household status considerations work in their favor. For stay-at-home wives, on the contrary the third term is always negative and household status consideration works against them. The last term is always negative or at the most equal to zero, for working women, because $w_f \leq w_m$, and thus individual status considerations work against the women who choose to work. As stay-at-home wives are not part of the workforce, their relative standing in the society is not defined by it, which is reflected by the last term of the utility function $ln(1) = 0$.[7]

I define the function $f(\mu_i)$ as the difference between the utility when staying home and the utility when working of the woman with taste μ_i, where $\tilde{\mu}$ needs to be determined.

$$f(\mu_i) = \mu_i \alpha_1 C_h - \alpha_2 ln\left(\frac{w_m + w_f - pC_h}{w_m}\right) - \alpha_3 ln\left(\frac{w_m + w_f}{w_m + pC_h}\right) - \alpha_4 ln\left(\frac{w_f(1+\tilde{\mu})}{w_m + \tilde{\mu} w_f}\right) \quad (6a)$$

The cut-off $\tilde{\mu}$ that gives us the fraction of women that are in the labor force is the value of μ_i, such that $f(\tilde{\mu})$=0. (6b)

A woman whose preference for producing her own home goods is given by $\tilde{\mu}$ is indifferent between working and staying at home. If $f(\mu_i)$>0, a woman whose preference for producing her

[7] The utility function is structured so that stay-at-home women compare their status with working women, but not with working men.

own home goods is given by μ_i will prefer to stay at home rather than work, and therefore $\tilde{\mu} < \mu_i$. If $f(\mu_i)$<0 a woman whose preference for producing her own home good is given by μ_i will prefer to work than stay at home, and thus $\tilde{\mu} > \mu_i$. Depending on the parameter values of $C_h,\ p,\ w_m, w_f$ there could be zero, one or two solutions for $\tilde{\mu}$. Under some mild conditions[8] a uniquely economically interesting solution for $\tilde{\mu}$ can be derived (see Fig.1 and Appendix).

3.2 Exogenous female wages

Impact on female labor force participation

Proposition 1: *The share of women who work (*$\tilde{\mu}$*) increases with a reduction in the price of "home goods", p* ($\frac{\partial \tilde{\mu}}{\partial p} < 0$)*, a decrease in male wage,* w_m ($\frac{\partial \tilde{\mu}}{\partial w_m} < 0$)*, and an increase in the female wage,* w_f*,* ($\frac{\partial \tilde{\mu}}{\partial w_f} > 0$)*).*

All proofs are contained in the appendix. A decrease in the price of "home goods" will make women more likely to work because they pay less to acquire the "home goods" from the market, and it also improves their household income relative to that of the stay-at-home wives.

A decrease in the male wage leads to an increase in female labor participation for several reasons. Firstly, all households suffer a loss in income when the male wage declines, causing a decrease in the consumption of market goods, but because the household income of working wives is higher, their utility loss is lower. Secondly, because the male wage represents a smaller proportion of household income when wives work outside the home, working wives' household position improves relative to that of households with stay-at-home wives. And finally, working women's relative wage increases as male wages decrease, and female wages remain constant.

An increase in female wages makes women more likely to join the labor force because they expect to obtain greater utility when they can consume more market goods, their household income increases relative to that of the stay-at-home wives and their individual income increases relative to that of their male colleagues.

[8] The stability condition is $\alpha_1 C_h > \alpha_4 \frac{w_m - w_f}{w_m}$. Intuitively, this condition is satisfied if gender wage difference and the importance that women put on their relative position in the working force are not prohibitively high so that nobody joins the labor force, and the quantity and the importance of home goods in terms of welfare are high enough so that at least some women stay at home.

Corollary 1: *Female participation is more sensitive to changes in female wages than to changes in male wages.*

$$\left|\frac{\partial \tilde{\mu}}{\partial w_m}\right| < \left|\frac{\partial \tilde{\mu}}{\partial w_f}\right|.$$

Lemma 1: *The share of women who work* ($\tilde{\mu}$) *increases if relative household income concerns become more important for welfare,* ($\frac{\partial \tilde{\mu}}{\partial \alpha_3} > 0$) *and if relative individual wage concerns become less important*($\frac{\partial \tilde{\mu}}{\partial \alpha_4} < 0$).

Different types of relative concerns have different effects on female participation. An increase in the importance of relative household position makes women more likely to work because the household status consideration always works in the favor of women who choose to work. An increase in the importance of relative individual position makes women less likely to work as individual status considerations always work against the women who choose to work.

Impact on Income

Proposition 2: *A decrease in price of "home goods", p has a negative effect on the average wage* ($\frac{\partial \overline{Y^{ind}}}{\partial p} > 0$)*, and ambiguous effects on the average household income* ($\frac{\partial \overline{Y^{hh}}}{\partial p} <> 0$)*. An increase in w_m has a positive effect on the average wage* ($\frac{\partial \overline{Y^{ind}}}{\partial w_m} > 0$)*, and ambiguous effects on average household income* ($\frac{\partial \overline{Y^{hh}}}{\partial w_m} <> 0$) *. Finally, an increase in w_f has ambiguous effects on the average wage* ($\frac{\partial \overline{Y^{ind}}}{\partial w_f} <> 0$) *and a positive effect on the average household income* ($\frac{\partial \overline{Y^{hh}}}{\partial w_f} > 0$).

A decrease in p leads to an increase in female participation (Proposition 1). The average income of the household where the wife stays at home decreases. Depending on how many women join the labor force and the extent of their additional income (i.e., the difference between gained wages and the value of the "home goods" that they now purchase), the overall effect on the average household income could be either positive or negative. Additionally, as more women enter the labor force, the average wage in the economy decreases.

An increase in w_m leads to an increase in the income of each household. However, the same increase also leads to a decrease in female participation (Proposition 1) and thus there is an income loss for each household in which the wife switches to produce "home goods" whose value is less than the wage she had received in the workplace. Additionally, as fewer women remain in the labor force and the male wage increases, the average wage in the economy increases.

An increase in w_f leads to an increase in the income of each household with working women. It also leads to increased female participation (Proposition 1), and thus brings additional income due to the difference between the female wage and the value of the "home goods." As more women enter the labor force the average wage should decrease. Given that the wages of the women who were already working increased, however, the overall effect on the average wage in the economy is ambiguous.

As long as there is an interior solution for the fraction of women working (i.e., $\tilde{\mu} \in (0,1)$) I define the "perennial" working woman as the woman who would work regardless of whether the shock studied occur (i.e., $\mu_i = 0$). Similarly, I define the "perennial" stay-at-home wife as the woman who would stay at home regardless of whether the shock studied occurred (i.e., $\mu_i = 1$). Thus, all the results that refer to the "perennial" woman, either working or stay-at-home, refer to cases when there exist interior solutions for $\tilde{\mu}$ both before and after the economic shock.[9] I discuss the limiting cases, when $\tilde{\mu} = 0$ or $\tilde{\mu} = 1$, when they offer interesting insights.

Impact on Welfare

I analyze the welfare effects of different economic shocks, both at the individual level and at averages across the economy. One major factor that affects the welfare of the "average" working women is the selection of women into the labor force. Because women have heterogeneous and time-invariant preferences for producing their own "home goods," changes in the proportion and thus the composition of the women that are in the labor force have a significant impact on the average welfare. Thus, if more women drop out of the labor force, the effect on the welfare of the "average" working wife is more positive or less negative than the effect on the "perennial" working wife. If more women join the labor force, the effect on the welfare of the "average" working wife is more negative or less positive than on the "perennial" working wife.

Lemma 2*: The average welfare of stay-at-home wives is equal to each stay-at-home wife's welfare, and the average welfare of working wives depends on the fraction of women that work (*$\tilde{\mu}$*).*

[9] Because women in the model only live for one period, the "perennial" working or stay-at-home woman refers to women who share the same characteristics, μ_i, not to the same woman per se.

Lemma 3*:* *If all women work* ($\tilde{\mu} = 1$) *the change in welfare of each working woman is identical to the change in average welfare of working women.*

Proposition 3: *A decrease in the price of "home goods" (* p) *has a negative effect on the welfare of the stay-at-home wife and an ambiguous effect on the welfare of working wives*

The decrease in stay-at-home wives' welfare is related strictly to their household's relative standing. The value of the goods they produce declines, which in turn reduces both the absolute and relative value of their household and, as more women join the labor force, their relative household income deteriorates even more.

As the price of "home goods" decreases, the working wives can buy more market goods. Since the value produced by the stay-at-home wives decreases, working women's relative household standing improves. If participation rates remained constant, the working women would be unambiguously better off. However, as more women join the labor force the average household income in the economy might increase, and thus their relative position might deteriorate. As more women enter the labor force, the average wage in the economy decreases, and their relative wage improves.

Compared to the welfare of the "perennial" working woman, additional compositional effects are considered when examining the welfare of the average woman. The women who join the labor force as a consequence of the decrease in the price of "home goods" are women who suffer a stronger disutility from not producing their own "home goods." Overall these combined effects imply that the average working woman at a time when the price of "home goods" has decreased, who, I must emphasize, is different from the average working woman when p was higher, may be worse off as a consequence of a decrease in p under a wider set of parameters.

Using Lemma 3, it is apparent that if all women work both before and after the shock ($\tilde{\mu} = 1$) a decrease in the price of "home goods" has a consistently positive effect on the average utility of working wives.

Stevenson and Wolfers (2009) argue that the decrease they observe in women's happiness, which is contemporaneous with increased female participation rates, might be explained by women's exposure to new reference groups when they enter the labor force. Proposition 3 shows, however, that status concerns at the household level, not at the individual level, are the fundamental factor that might lead to a decrease in the happiness of the average working woman. If this status concern is not present, stay-at-home wives' utility would remain constant when the price of "home goods" changes because their utility would depend only on the level of goods that they produce, not their price. In this case an increase in women's participation in the context of a decrease in the price of "home goods" would be possible only if working women's utility were higher than the utility of wives who stayed at home, thus a higher utility than what they experienced before, when the price of "home goods" was higher.

Proposition 4: *An increase in the male wage (w_m) has a positive effect on the welfare of the stay-at-home wife and an ambiguous effect on the welfare of the working wives.*

Stay-at-home wives enjoy an increase in welfare because they have access to more market goods when their husbands' wages are higher. Stay-at-home wives' relative household position, and thus their welfare, improves for two reasons: their husband's income represents a bigger share of their household worth and, as female participation decreases (Proposition 1), the average household income becomes closer to their household income.

Working wives enjoy more market goods when their husbands' wages increase, but working women's relative status as individual workers deteriorates as the gender gap widens and there are fewer women in the work force, which pushes the average wage up. The effect on working women's relative household position is ambiguous: on one hand, their husbands' incomes represent a smaller share of their household worth and their household status deteriorates, but on the other hand, as fewer women work, their household income is proportionally higher than the average in the economy.

The women who drop out of the labor force as a consequence of an increase in the male wage are those who suffer a stronger disutility from not producing their own "home goods." Thus, the average working woman is better off than the "perennial" working woman if male wages increase.

Proposition 5: *An increase in the female wage has a negative effect on the welfare of the stay-at-home wife and ambiguous effects on the welfare of the working wife.*

The decrease in stay-at-home wives' welfare is related strictly to their relative household standing. The increased wages that working women earn diminish the relative household worth of stay-at-home wives and, as more women join the labor force, stay-at-home wives' relative household standing decreases even more.

As the female wages increase, working wives can buy more market goods. Working women's individual standing improves as the gender wage gap narrows, and the increased female participation rate brings the average wage in the economy closer to their own wage. The effect on working women's relative household income is ambiguous. Their absolute household income increases as their wage increases, but as more women join the labor force the average household income in the economy increases as well, and thus their relative position might deteriorate. Even though most factors indicate that an increase in the female wage would improve the welfare of the "perennial" working woman, the overall effect on welfare is surprisingly ambiguous.

Women who join the labor force as a consequence of the increase in the female wage are women who suffer a stronger disutility from not producing their own "home goods", and thus the effect on the welfare on average working women is even more likely to be negative.

If all women already work (i.e., $\tilde{\mu} = 1$), an increase in w_f has, however, an unambiguously positive effect on the welfare of the working wife.

3.3. Part-time work

In this section I allow for the possibility of part-time work, so that women can spend λ time producing the home good and $1 - \lambda$ working for a wage outside the home. I derive the utility function for women who work part-time, substituting for the household implicit income and quantity of market goods consumed in a household where the woman works part-time[10].

$$u_i = \lambda\alpha_1 C_h + (1-\lambda)(1-\mu_i)\alpha_1 C_h + \alpha_2 \ln\big(w_m + (1-\lambda)(w_f - pC_h)\big) + \alpha_3 ln\left(\frac{w_m + \lambda pC_h + (1-\lambda)w_f}{\overline{Y^{hh}}}\right) + \alpha_4 \ln\left(\frac{w_f(1+\tilde{\mu}_{pt})}{w_m + \tilde{\mu}_{pt} w_f}\right) \quad (7)$$

Similarly to (6a), I define the function $g(\mu_i)$ as the difference between the utility when staying home and the utility when working part-time of the woman with taste μ_i, where $\tilde{\mu}_{pt}$ needs to be determined.

$$g(\mu_i) = \mu_i(1-\lambda)\alpha_1 C_h - \alpha_2 ln\left(\frac{w_m + (1-\lambda)(w_f - pC_h)}{w_m}\right) - \alpha_3 ln\left(\frac{w_m + \lambda pC_h + (1-\lambda)w_f}{w_m + pC_h}\right) - \alpha_4 \ln\left(\frac{w_f(1+\tilde{\mu}_{pt})}{w_m + \tilde{\mu}_{pt} w_f}\right) \quad (8.a)$$

[10] I assume that women who work part-time do not pay any penalty in terms of wage benefits, thus their individual worker status is identical to that of women that work full-time, as they compare their wages based on wage per unit of time worked.

The cut-off $\tilde{\mu}_{pt}$ that gives us the fraction of women that would be in the labor force part-time, if full-time were not available is the value of μ_i, such that $g(\tilde{\mu}_{pt})$=0.[11] (8.b)

From Section 3.1, $\tilde{\mu}$ is the fraction of women that would work full-time if part-time positions were not available.

If both part-time and full-time work are available in the economy, since women who work on a part-time basis consume less of the market goods, and their household status is lower, their preference for producing their own home goods is always higher than that of women that work full-time $\mu_i^{pt} > \mu_i^{ft}$. In this case, there are several potential equilibria: female labor force participation is zero ($\tilde{\mu} = 0,\ \tilde{\mu}_{pt} = 0$), women take only full-time positions if $\tilde{\mu}_{pt} \leq \tilde{\mu}$, (where $\tilde{\mu} > 0, \tilde{\mu}_{pt} \geq 0$) or they take both full-time and part-time positions. In the third scenario $\tilde{\mu}_{pt} > \tilde{\mu}$, and overall female labor force participation is given by $\tilde{\mu}_{pt}$. Using (7) and (1') I can derive $\tilde{\mu}_{ft}$, such that a woman whose preference for producing her own home goods is given by $\tilde{\mu}_{ft}$ is indifferent between working part-time or full-time.

$$\tilde{\mu}_{ft} = \frac{1}{\lambda\alpha_1 C_h}\left(\alpha_2 ln\left(\frac{w_m+w_f-pC_h}{w_m+(1-\lambda)(w_f-pC_h)}\right)+\alpha_3 ln\left(\frac{w_m+w_f}{w_m+\lambda pC_h+(1-\lambda)w_f}\right)\right) \qquad (9)$$

Women who choose to work full-time have $\mu_i \leq \tilde{\mu}_{ft}$, and women that have $\tilde{\mu}_{ft} < \mu_i \leq \tilde{\mu}_{pt}$ work part-time. If $\tilde{\mu}_{ft} < \tilde{\mu}$, overall female labor force participation increases thanks to the availability of part-time work, but some women that would have worked full-time end up working part-time. This result is consistent with the findings of Blau and Kahn (2013), which show that the possibility of part-time jobs can facilitate the labor force entry of less career-oriented women, but it could also encourage women who would otherwise have a stronger labor force commitment to take part-time jobs.

The availability of part-time work affects the welfare of women in an ambiguous fashion. Focusing on the case where there is an increase in overall female participation associated with a decrease in the women working full-time, the welfare of women is affected through the change in their household status, due to the change in average household income $\overline{Y^{hh}} = w_m + \tilde{\mu}_{ft} w_f +$

[11] Depending on the parameter values of $C_h,\ p,\ w_m, w_f$ and λ there could be zero, one or two solutions for $\tilde{\mu}_{pt}$. If there exists a stable interior solution such that $f(\tilde{\mu}_{pt}) = 0$, the stability condition is derived as $(1-\lambda)\alpha_1 C_h > \alpha_4 \frac{w_m - w_f}{w_m}$.

$\left(\tilde{\mu}_{pt} - \tilde{\mu}_{ft}\right)\left[\lambda p C_h + (1-\lambda) w_f\right] + \left(1 - \tilde{\mu}_{pt}\right) p C_h$. The decrease in full-time female employment leads to a decrease in average household income, whereas the increase in overall female employment leads to an increase in average household income. Given the ambiguous effects on the average household income, the effect of the availability of part-time work on the welfare of either working or stay-at-home women is ambiguous as well.

The analyses in Section 3.2 and 3.3 were made assuming that female wages are exogenous. In Section 3.4, I assume that female wages are endogenous.

3.4 Endogenous female wages

In this section, I assume that female wage is a function of the male wage and the past female labor force participation rate. I make this assumption because technological advances in the marketplace would affect the demand for labor in general, be it male or female. I assume that female and male labor are perfect substitutes. However, given the different historical trajectories in the labor market, women are at a disadvantage in obtaining the same jobs that men do, and thus are paid a lower salary. If all women were in the labor market, then female and male wages would be identical. To capture these effects, I also introduce time transcripts in the female wage equation:

$$w_{f,t} = \theta w_m + (1-\theta)\tilde{\mu}_{t-1}^{\beta} w_m \tag{10}$$

Where β can be greater than, less than, or equal to one. The three different cases will have different implications on the hysteresis associated with married women's entrance into or exit from the labor market.

In steady state $\tilde{\mu}_{t-1} = \tilde{\mu}_t$, and equilibrium wage and female participation $(\widehat{w}_f, \hat{\mu})$ can be derived using (6b) and (10). The slope as given by (10) is $\frac{dw_{f,t}^{Ld}}{d\tilde{\mu}_{t-1}} = \beta(1-\theta)\tilde{\mu}_{t-1}^{\beta-1} w_m > 0$ and its curvature depends on β so that: $\frac{d^2 w_{f,t}^{Ld}}{d\tilde{\mu}_{t-1}^2} = \beta(\beta-1)(1-\theta)\tilde{\mu}_{t-1}^{\beta-2} w_m \leq (>) 0$ if $\beta \leq (>) 1$. Given that the female wage as given by both (6b) and (10) is upward-sloping in $\tilde{\mu}_t$ (see also Fig 2a, b and c), there exists at most one stable solution for $(\widehat{w}_f, \hat{\mu})$ under some mild stability conditions (see Appendix). If the starting point is a disequilibrium such that $w_{f,t}$ as given by (10) is higher than $w_{f,t}$ as given by (6b), a higher $w_{f,t}$ leads to an increase in $\tilde{\mu}_t$ (Proposition 1), which leads to an increase in $w_{f,t}$ (Eq. 7), and after several iterations equilibrium is reached. If the starting point is a disequilibrium such that $w_{f,t}$ as given by (10) is lower than $w_{f,t}$ as given by (6b), the lower $w_{f,t}$

leads to a decrease in $\tilde{\mu}_t$ (Proposition 1), which leads to a decrease in $w_{f,t}$ (10), and after several iterations equilibrium is reached.

3.4.1 Dynamics and hysteresis

I first analyze the dynamic effects that a decrease in the price of "home goods" p has on female participation, $\hat{\mu}$ and the female wage, $\hat{w}_f$. A reduction in the price of "home goods" p, encourages women to join the labor force (Proposition 1). The increase in the share of women who work leads to an increase in female wages, which also has a positive effect on labor participation. Thus, the adjustment to higher levels of female participation and wages takes place monotonically.

Secondly, I analyze the dynamic effects that an increase in the male wage, w_m has on female participation, $\hat{\mu}$ and wage, $\hat{w}_f$. The increase in the male wage leads to an increase in the female wage, which positively affects the female participation rate. The increase in female participation in subsequent generations leads to even more increases in female wages, which encourages even more women to work. This dynamic process diffuses throughout several generations of women and will lead eventually to a new steady-state equilibrium where female wages and labor force participation are higher than before.

Due to the existence of two possible equilibria, one stable and one unstable, the entrance and exit trajectories in the labor force as a consequence of changes in male wage and the price of "home goods" might not be symmetrical. The shape of the labor demand function, whether it is a strictly convex or concave function of past female participation, which is given by the values of β in relationship to one, affects the type of hysteresis that could arise. The following sections analyze each case in detail.

Hysteresis when $\boldsymbol{\beta} > 1$

If there are two solutions for $\hat{\mu}$ in the interval [0,1] the stable one is the smaller one (Fig. 2a). In this case, hysteresis is likely if the economy experiences full female participation.

Fig. 3a, b, c and d are graphical representations of different cases of hysteresis when $\boldsymbol{\beta} > 1$. Starting from an interior solution equilibrium, changes in any of the exogenous variables lead to the expected variation in the endogenous variables (Proposition 6). There exist certain thresholds w_m^0 and p^0 such that if $w_m < w_m^0$ and respectively $p > p^0$ women's participation is equal to zero, $\hat{\mu} = 0$ and $\hat{w}_f = \theta w_m$. However, if the economy starts from an equilibrium with full female participation, $\hat{\mu} = 1$ there are two cases: (1) full participation is irreversible and (2) full participation is reversible as long as there exist w_m^* and respectively $p*$ at which the system has only one stable and no unstable equilibrium for $\hat{\mu} \in [0,1]$. In case (2) as long as $w_m > w_m^*$ *(p<p*)* female participation is 100 percent and only once w_m drops below w_m^* or in the case of the price of "home goods" p rising above $p*$ women's participation will strongly decrease to an equilibrium at a much lower participation rate.

Fig. 3a, b, c and d show that for the same value of w_m (p) between w_m^* (p^{**}) and w_m^{**} (p^*) there is full female participation or not depending on the history. Within these intervals, the same variation in the male wage (the price of "home goods") can have therefore dramatically different effects on female participation and wages.

Hysteresis when $\boldsymbol{\beta} \leq \mathbf{1}$

If there are two solutions for $\hat{\mu}$ in the interval [0,1] the stable one is the larger one (Fig. 2b and c). In this case, hysteresis is likely if the economy starts from zero female participation.

Fig. 4a, b, c and d are the graphical representations of different cases of hysteresis when $\boldsymbol{\beta} \leq \mathbf{1}$. Starting from an interior solution equilibrium, changes in any of the exogenous variables lead to the expected variation in the endogenous variables (Proposition 6). There exist certain thresholds w_m^1 and p^1 such that if $w_m > w_m^1$ and respectively $p < p^1$ there is full female participation, $\hat{\mu} = 1$ and $\widehat{w}_f = w_m$. However, starting from an equilibrium in which female participation is null $\hat{\mu} = 0$ there are two cases that can follow: (1) null participation rate is immutable and (2) the null participation rate is not permanent, as long as there exist w_m^* and respectively $p*$ at which the system has only one stable and no unstable equilibrium for $\hat{\mu} \in [0,1]$. In case (2) as long as $w_m < w_m^*$ *(p>p*)* there is no female participation, $\hat{\mu} = 0$, but if $w_m \geq w_m^*$ *(p ≤p*)* female participation jumps to the equilibrium level that is implied by the parameters of the model and has no historical influence.

Fig. 4a, b, c and d show that for the same value of w_m (p) between w_m^{**} (p^*) and w_m^* (p^{**}) there is female participation or not depending on the starting equilibrium point. Within these intervals the same variation in the male wage (price of "home goods") can have dramatically different effects on female participation and wages.

3.4.2 Occupations

The dynamic portion of the model and the hysteresis associated with it is especially relevant in the context of women entering different occupations.

Let's consider two different occupations: due to historical reasons, female participation is positive in Occupation 1, $\hat{\mu}_1 > 0$, and female participation is null in Occupation 2, $\hat{\mu}_2 = 0$. If $p \in (p^*, p^{**})$, the interval that allows for multiple equilibria, a decrease in p would lead to increased participation in Occupation 1, but it would have no effect on female participation in Occupation 2. Similarly, if $w_m \in (w_m^{**}, w_m^*)$, positive shocks to w_m lead to increases in labor participation in Occupation 1, but no changes in female participation in Occupation 2.

Once $w_m \geq w_m^*$ ($p \leq p^*$), female participation in Occupation 2 reacts to shocks to w_m and p in a similar manner to Occupation 1 (see also Fig 3a,b,c).

Factors that lead to increased female labor force participation would thus channel women first in the occupations in which they were historically working, leading to a decrease in gender wage gap in those occupations. The female participation and wage gaps in the male dominated occupations might take longer to be affected. Shocks that lead to increased overall female participation can thus lead to no changes in female participation and wages in male dominated professions, concurrent with a small reduction in overall gender gap. Productivity shocks that affect disproportionately male dominated occupations might even lead to increased female participation concurrent with an increase in the overall gender gap.

Blau and Kahn (2017) show that 49% of the wage gap can be explained by occupational differences between men and women. They find that even though during 1980-2010 women upgraded their occupations, which lead to a narrowing of the gender wage gap, disproportionately increased returns to male dominated occupations had the opposite effect. Blau, Brummund, and Liu (2013) develop an occupational segregation index and show that there was a decrease in segregation from 1970 to 2009, but also that the occupational difference between women and men remains large. The hysteresis effects derived in this section provide a plausible explanation for why women have not entered occupations in a proportional manner[12], and why the gender wage gap is more persistent than aggregate models would predict.

[12] History dependent path is not the only way to generate different occupational choices for men and women. Erosa, Fuster, Kambourov, and Rogerson (2017) develop a model of occupational choice and labor supply and find that women are disproportionately represented in occupations with low mean hours and wages. In their model the gender asymmetry in market outcomes such as occupational choice and gender wage gap is caused by the gender difference in time allocated to home production.

3.4.3 Comparative statics results for interior solutions

Impact on female labor force participation

Proposition 6: *A reduction in the price of "home goods"* (p) *leads to an increase in the share of women who work (* $\frac{\partial \hat{\mu}}{\partial p} \leq 0$*), as well as the female wage (* $\frac{\partial \hat{w}_f}{\partial p} \leq 0$*) and a decrease in the gender wage gap (*$\frac{\partial (\hat{w}_f - w_m)}{\partial p} \leq 0$*). An increase in the male wage,* w_m *leads to an increase in female participation* $\left(\frac{\partial \hat{\mu}}{\partial w_m} \geq 0\right)$*, an increase in the female wage (*$\frac{\partial \hat{w}_f}{\partial w_m} \geq 0$*), and ambiguous effects on the gender wage gap (*$\frac{\partial (\hat{w}_f - w_m)}{\partial w_m} <> 0$*)* .

A decrease in the price of "home goods" leads to an increase in the share of women who work (Proposition 1) and given (10) to an increase in the female wage in equilibrium. As male wages remain constant, the gender gap decreases.

An increase in the male wage instantaneously leads to an increase in the female wage, which would encourage women to join the labor force (Proposition 1). Simultaneously, the increase in the male wage, by increasing the utility of stay-at-home wives relative to working wives, discourages women from joining the labor force (Proposition 1). In this scenario, with the conditions attached to the existence of a stable equilibrium, the first effect on female participation always dominates.

What is interesting about these results is that a change in male wages has the opposite effect on female participation than it did when female wages were exogenous. Male wages are directly affecting female wages and participation in the endogenous model, and female wages are subsequently affecting women's participation. This indirect effect on participation turns out to be more important than the direct one if a stable equilibrium is to be reached. Intuitively, this result is due to the fact that abstracting from the feedback effect that participation has on female wages, female participation is much more sensitive to changes in female wages than to changes in male wages (Corollary 1).

Lemma 4: *The share of women who work* $(\hat{\mu})$ *and the female wage* $(\hat{w}_f)$ *both increase if relative household concerns become more important for welfare (*$\frac{\partial \hat{\mu}}{\partial \alpha_3} > 0$; $\frac{\partial \hat{w}_f}{\partial \alpha_3} > 0$*) and if relative individual wage concerns become less important (*$\frac{\partial \hat{\mu}}{\partial \alpha_4} < 0$; $\frac{\partial \hat{w}_f}{\partial \alpha_4} < 0$).

An increase in the importance of relative household position makes women more likely to work, whereas an increase in the importance of relative individual position makes women less likely to work (Lemma 1). Relative concerns affect female wage only through their effect on labor participation.

Impact on Income

Proposition 7: *A decrease in* p *has an ambiguous effect on average household income* ($\frac{\partial \overline{Y^{hh}}}{\partial p} <> 0$) *and the average wage* ($\frac{\partial \overline{Y^{ind}}}{\partial p} <> 0$). *An increase in* w_m *has a positive effect on average household income* ($\frac{\partial \overline{Y^{hh}}}{\partial w_m} > 0$) *and the average wage* $\left(\frac{\partial \overline{Y^{ind}}}{\partial w_m} > 0\right)$.

What is remarkable compared to Proposition 2 is that an increase in w_m, given that it is associated with an increase in $\widehat{w}_f$ and $\hat{\mu}$, has a clear positive effect on average household income, as opposed to an effect on the average individual income only. Also contrasting with Proposition 2, a decrease in p, by increasing female participation, affects $\widehat{w}_f$ positively, and these two effects combined lead to an ambiguous effect on the average individual wage.

Impact on Welfare

Proposition 8 *A decrease in the price of "home goods"* (p) *has a negative effect on the welfare of the stay-at-home wife and an ambiguous effect on the welfare of the working wife.*

The decrease in welfare enjoyed by stay-at-home wives as a consequence of a reduction in the price of "home goods" is related strictly to their household relative standing. Incorporating the fact that the female wage also increases, the reduction in welfare is more significant than if female wages were exogenous.

Similarly, the effect on the welfare of working women is complicated by the endogeneity of the female wage. An increase in female wages as a result of increased participation introduces additional sources of ambiguity with respect to the effect on women's welfare, as Proposition 4 indicates. The main source of negativity in working women's welfare comes from relative concerns at household level. As participation increases, the woman that was working even before the shock will face a decline in her household relative income, which might counteract all the positive effects brought by her improved individual standing and her household's increased ability to buy market goods.

The additional compositional effects brought about by increased labor participation would tend to lower the welfare of the average working woman compared to the "perennial" one.

If all women work ($\hat{\mu} = 1$) a decrease in p has an unambiguously positive effect on the welfare of the working wife as in Proposition 3, as neither female participation nor female wages change.

Proposition 9: *An increase in the male wage* (w_m) *has an ambiguous effect on the welfare of both the stay-at-home and working wife.*

An increase in male wages leads to an improvement in stay-at-home wives' ability to buy market goods and improve their relative status (Proposition 4). Because this change in the male wage leads to an increase in the female wage as well (Proposition 6), which has a negative effect on the welfare of stay-at-home wives (Proposition 5), the overall welfare effect is ambiguous.

4. Conclusions

Female labor participation increased dramatically over the second half of the 20th century in most industrialized countries. Some of the factors that led to this development, by most objective criteria, should have also led to increases in women's welfare. Yet Stevenson and Wolfers (2009) show that during this time, women's happiness levels declined, and this trend can be found across demographic groups and industrialized countries.

In this paper, I use relative status concerns at both household and individual level to explain how decreases in the price of "home goods" relative to female wages and increase in women's wages (decreases in gender wage gaps) that contributed to increased labor participation of married women affected female welfare, as well as average household income and wage in the economy. An increase in female wages will always be associated with an increase in the average household income, but not necessarily with an increase in the average wage in the economy depending on the elasticity of female participation with respect to their wages. An increase in male wages leads to an increase in the average wage in the economy, but not necessarily of the average household income depending on the relationship between female and male wages.

Endogenizing female wage and expressing it as a positive function of male wage and past female participation rates leads to multiple equilibria. Depending on the curvature of the female wage as a function of past participation rate, hysteresis appears in association with married women's entrance into or exit from the labor market. Women not entering occupations in a proportional manner, as predicted by the hysteresis effect in the model, is a key component of occupational differences and wage gaps between men and women.

I analyze the welfare effects of different economic shocks, at the individual level and averages across the economy for both the stay-at-home and working wives. One major factor that affects the welfare of the "average" working women is compositional. Because women have heterogeneous and time-invariant preferences for producing their own "home goods," changes in the proportion and thus the composition of the women that are in the labor force have a significant impact on the average welfare. Thus, if more women drop out of the labor force, the effect on the welfare of the "average" working wife is more positive or less negative than the effect on the "perennial" working wife. If more women join the labor force, the effect on the welfare of the "average" working wife is more negative or less positive than on the "perennial" working wife. Availability of part-time work leads to increased female labor force participation, but some women that would have worked full-time end up working part-time. The welfare effect of this availability is ambiguous.

A reduction in the price of "home goods" decreases the welfare of the stay-at-home wives and has an ambiguous effect on the welfare of the working wives, depending on how much they care about their household's relative income. If female wages are independent from male wages, an increase in female wages has positive effects and a decrease in male wages has ambiguous effects on the welfare of working wives, even though in both scenarios female participation rates increase. The welfare of stay-at-home wives is affected negatively. If female wage, however, is endogenous, a decrease in the male wage is associated with a decrease in the female wage and participation rate, and the overall welfare effects on both the stay-at-home and working wives are ambiguous.

The different experiments performed in the paper show that factors leading to increased female participation are usually associated with decreases in the welfare of stay-at-home wives, but surprisingly are not always associated with increases in the welfare of working wives. Thus, policies designed to increase female participation would not necessarily lead to an increase in the working women's welfare, even if they would enjoy higher wages associated with higher participation.

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Figures

Fig. 1

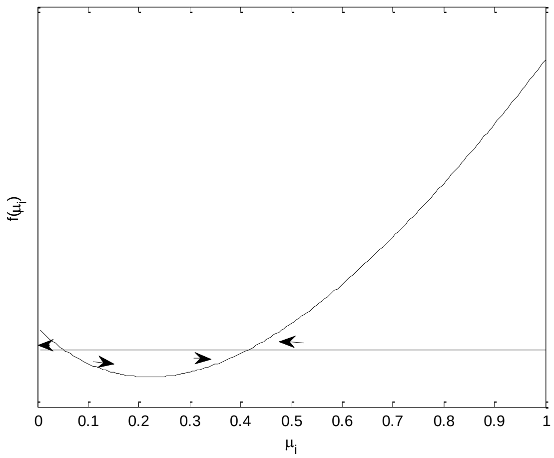


Fig. 2.a

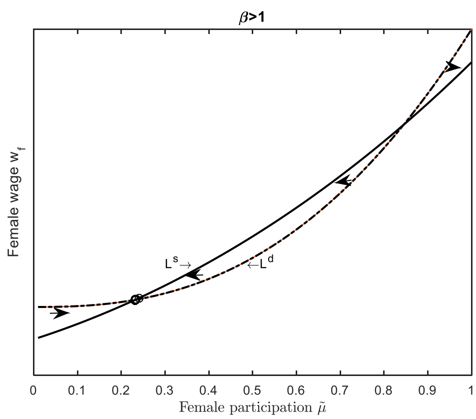


Fig. 2.b

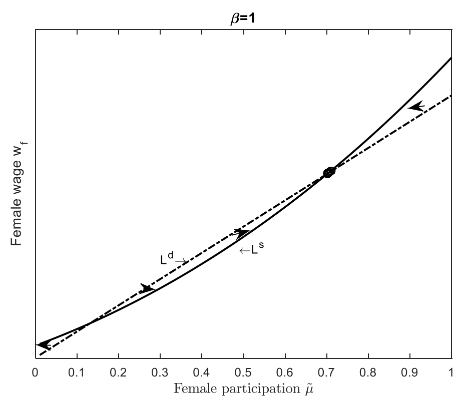
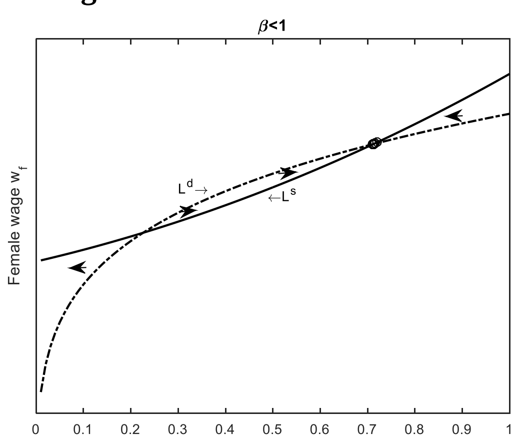


Fig. 2.c



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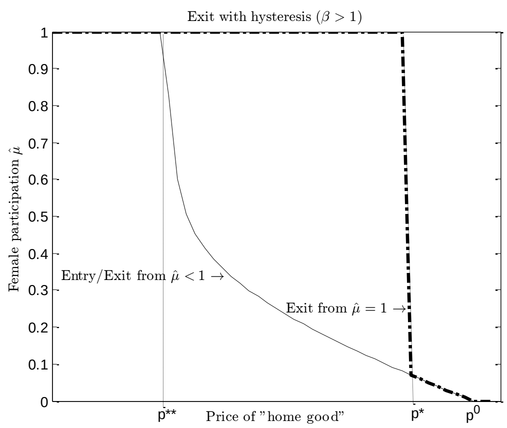


Fig 3.b.

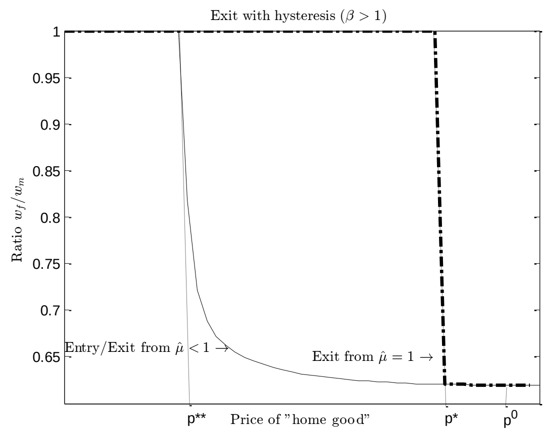


Fig. 3.c.

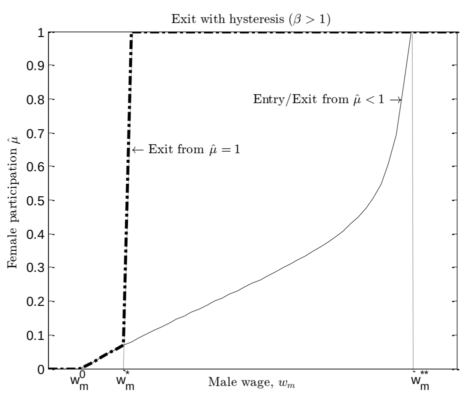


Fig 3.d

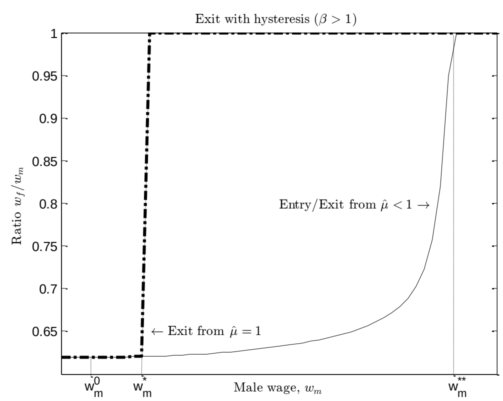


Fig. 4.a

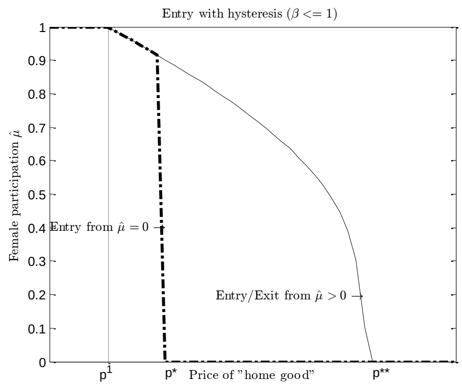


Fig. 4.b

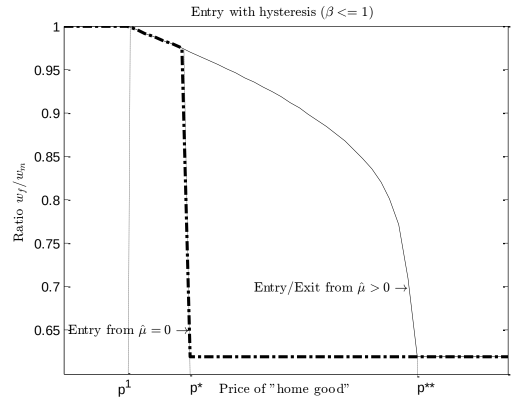


Fig. 4.c

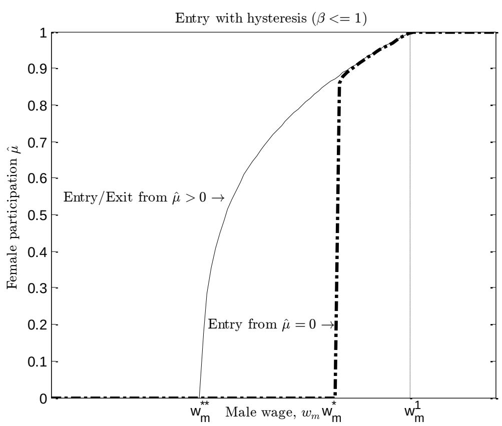
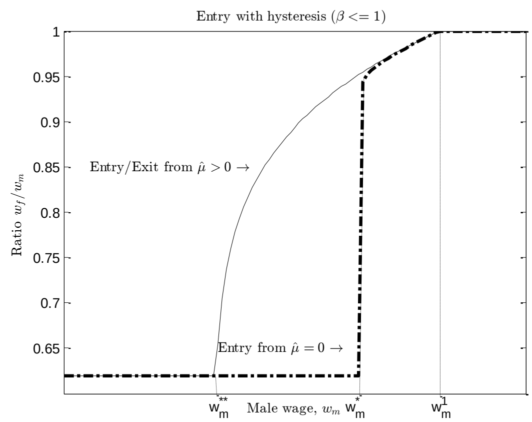


Fig. 4.d



Appendix

Stability condition for $\tilde{\mu}$

From (6a) I define the function $f(\mu_i)$ as the difference between the utility when staying home and the utility when working of the woman with taste μ_i.

Thus, if there exists a stable interior solution such that $f(\tilde{\mu})=0$, then $f(\tilde{\mu}-\epsilon)<0$ and $f(\tilde{\mu}+\epsilon)>0$. The stability condition in this case is $(f'(\mu)>0|\mu=\tilde{\mu})$, thus $\alpha_1 C_h - \alpha_4 \frac{w_m - w_f}{(w_m+\tilde{\mu}w_f)(1+\tilde{\mu})} > 0$. (1A)

Let $g(\tilde{\mu}) = \alpha_1 C_h - \alpha_4 \frac{w_m - w_f}{(w_m+\tilde{\mu}w_f)(1+\tilde{\mu})}$. Given that $g'(\tilde{\mu}) = \alpha_4 (w_m - w_f) \frac{w_m + w_f + 2\tilde{\mu}w_f}{\left((w_m+\tilde{\mu}w_f)(1+\tilde{\mu})\right)^2} > 0$,

Since $\tilde{\mu} \in [0\ 1]$, $g(\tilde{\mu})>0$ iff $g(0)>0$ and therefore $\alpha_1 C_h > \alpha_4 \frac{w_m - w_f}{w_m}$. Intuitively, this stability condition tells us that there exist a stable unique equilibrium for female labor force participation if gender wage difference, and the importance that women put on their relative position in the working force are not prohibitively high relative to the quantity and the importance of home good in the utility function.

Proof of Proposition 1

Applying the implicit equation theory for (6b), using (5a) and (5b), and imposing the stability condition (1A):

$$\frac{d\tilde{\mu}}{dp} = \frac{-C_h\left(\alpha_2 \frac{w_m}{w_m+w_f-pC_h} + \alpha_3 \frac{1}{w_m+pC_h}\right)}{\alpha_1 C_h - \alpha_4 \frac{w_{m,t}-w_{f,t}}{(w_m+\tilde{\mu}w_f)(1+\tilde{\mu})}} < 0 \qquad \text{(2A.a)}$$

$$\frac{d\tilde{\mu}}{dw_m} = -\frac{\alpha_2 \frac{w_f - pC_h}{w_m(w_m+w_f-pC_h)} + \alpha_3 \frac{w_f - pC_h}{(w_m+w_f)(w_m+pC_h)} + \alpha_4 \frac{1}{(w_m+\tilde{\mu}w_f)}}{\alpha_1 C_h - \alpha_4 \frac{w_m - w_f}{(w_m+\tilde{\mu}w_f)(1+\tilde{\mu})}} < 0 \qquad \text{(2A.b)}$$

$$\frac{d\tilde{\mu}}{dw_f} = \frac{\frac{\alpha_2}{w_m+w_f-pC_h} + \frac{\alpha_3}{(w_m+w_f)} + \alpha_4 \frac{w_m}{w_f} \frac{1}{(w_m+\tilde{\mu}w_f)}}{\alpha_1 C_h - \alpha_4 \frac{w_m - w_f}{(w_m+\tilde{\mu}w_f)(1+\tilde{\mu})}} > 0 \qquad \text{(2A.c)}$$

Proof of Corollary 1

Direct consequence of (5a), (5b) applied to (2A.b) and (2A.c)

Proof of Lemma 1

Applying the implicit equation theory for (6b), using (5a) and (5b), and imposing the stability condition (1A):

$$\frac{\partial \tilde{\mu}}{\partial \alpha_3} = \frac{ln\left(\frac{w_m+w_f}{w_m+pC_{\hbar}}\right)}{\alpha_1 C_{\hbar} - \alpha_4 \frac{w_m-w_f}{(w_m+\tilde{\mu}w_f)(1+\tilde{\mu})}} > 0 \tag{2A.d}$$

$$\frac{\partial \tilde{\mu}}{\partial \alpha_4} = \frac{ln\left(\frac{w_f(1+\tilde{\mu})}{w_m+\tilde{\mu}w_f}\right)}{\alpha_1 C_{\hbar} - \alpha_4 \frac{w_m-w_f}{(w_m+\tilde{\mu}w_f)(1+\tilde{\mu})}} < 0 \tag{2A.e}$$

Proof of Proposition 2

Using (3) and (2A.a):

$$\frac{\partial \overline{Y^{hh}}}{\partial p} = \frac{d\tilde{\mu}}{dp}\left(w_f - pC_{\hbar}\right) + (1-\tilde{\mu})C_{\hbar} <> 0 \tag{3A.a}$$

Using (4) and (2Aa):

$$\frac{\partial \overline{Y^{ind}}}{\partial p} = \frac{\frac{\partial \tilde{\mu}}{\partial p}(w_f - w_m)}{(1+\tilde{\mu})^2} > 0 \tag{3A.b}$$

Using (3) and (2A.b.):

$$\frac{\partial \overline{Y^{hh}}}{\partial w_m} = \frac{\partial \tilde{\mu}}{\partial w_m}\left(w_f - pC_{\hbar}\right) + 1 <> 0 \tag{3A.c}$$

Using (4) and (2Ab):

$$\frac{\partial \overline{Y^{ind}}}{\partial w_m} = \frac{\frac{\partial \tilde{\mu}}{\partial w_m}(w_f - w_m)+1+\tilde{\mu}}{(1+\tilde{\mu})^2} > 0 \tag{3A.d}$$

Using (3) and (2Ac):

$$\frac{\partial \overline{Y^{hh}}}{\partial w_f} = \frac{\partial \tilde{\mu}}{\partial w_f}\left(w_f - pC_{\hbar}\right) + \tilde{\mu} > 0 \tag{3A.e}$$

Using (4) and (2Ac):

$$\frac{\partial \overline{Y^{ind}}}{\partial w_f} = \frac{\frac{\partial \tilde{\mu}}{\partial w_f}(w_f - w_m)+\tilde{\mu}}{(1+\tilde{\mu})^2} <> 0 \tag{3A.f}$$

Proof of Lemma 2

As stay-at-home wives produce their own "home goods" using (1') it is inferred that their welfare is equal such that:

$Average\ Welfare\ \bar{s} = \ Welfare\ \bar{s}$

As working wives buy their "home" good and have heterogeneous preferences for producing it themselves, using (1') and (6) their average welfare can be written as:

$Average\ Welfare\ \bar{w} = \alpha_1 C_{\hbar}\left(1 - \frac{\tilde{\mu}}{2}\right) + \alpha_2 ln\left(w_m + w_f - pC_{\hbar}\right) +$

$\alpha_3 ln\left(\frac{w_m+w_f}{w_m+\tilde{\mu}w_f+(1-\tilde{\mu})pC_{\hbar}}\right) + \alpha_4 ln\left(\frac{w_f(1+\tilde{\mu})}{w_m+\tilde{\mu}w_f}\right)$

Proof of Lemma 3

$\frac{dAverage\ Welfare\ \bar{w}}{dx} = \frac{d\alpha_1 C_{\hbar}}{dx} + \frac{d\alpha_2 ln\left(w_m+w_f-pC_{\hbar}\right)}{dx} + \frac{d\alpha_3 ln\left(\frac{w_m+w_f}{w_m+\tilde{\mu}w_f+(1-\tilde{\mu})pC_{\hbar}}\right)}{dx} +$

$\frac{d\alpha_4 ln\left(\frac{w_f(1+\tilde{\mu})}{w_m+\tilde{\mu}w_f}\right)}{dx} - \alpha_1 C_{\hbar}\frac{1}{2}\frac{d\tilde{\mu}}{dx} = \frac{d\ Welfare\ \bar{w}(\mu_i)}{dx}$ for any μ_i if $\tilde{\mu} = 1$ and thus $\frac{d\tilde{\mu}}{dx} = 0$

Given that $C_{\hbar}$ is exogenous, $\frac{dC_{\hbar}}{dx} = 0$.

Proof of Proposition 3

Using (1'), (5a), and (2Aa):

$\frac{d\ Welfare\ \bar{s}}{dp} = \frac{\alpha_3}{\left(w_m+\tilde{\mu}w_f+(1-\tilde{\mu})pC_{\hbar}\right)}\left[C_{\hbar}\frac{w_m(1+\tilde{\mu})+w_f\tilde{\mu}+pC_{\hbar}}{\left(w_m+pC_{\hbar}\right)} - \frac{d\tilde{\mu}}{dp}\left(w_f - pC_{\hbar}\right)\right] > 0$

Using (1'), (5a), (5b) and (2Aa):

$\frac{dWelfare\ \bar{w}}{dp} = -\frac{\alpha_2 C_{\hbar}}{\left(w_m+w_f-pC_{\hbar}\right)} - \frac{\alpha_3(1-\tilde{\mu})C_{\hbar}}{w_m+\tilde{\mu}w_f+(1-\tilde{\mu})pC_{\hbar}} - \frac{d\tilde{\mu}}{dp}\left[\frac{\alpha_3(w_f-pC_{\hbar})}{w_m+\tilde{\mu}w_f+(1-\tilde{\mu})pC_{\hbar}} - \frac{\alpha_4\left(w_m-w_f\right)}{\left(w_m+\tilde{\mu}w_f\right)(1+\tilde{\mu})}\right] <$

> 0

If $\frac{d\tilde{\mu}}{dp} = 0$ then $\frac{dWelfare\ \bar{w}}{dp} < 0$

Using Lemma 3, assumptions (5a) and (5b), and (2Aa):

$\frac{dAverage\ Welfare\ \bar{w}}{dp} = -\alpha_2\frac{C_{\hbar}}{w_m+w_f-pC_{\hbar}} - \alpha_3\frac{(1-\tilde{\mu})C_{\hbar}}{w_m+\tilde{\mu}w_f+(1-\tilde{\mu})pC_{\hbar}} -$

$\frac{d\tilde{\mu}}{dp}\left[\frac{1}{2}\alpha_1 C_{\hbar} + \alpha_3\frac{w_f-pC_{\hbar}}{w_m+\tilde{\mu}w_f+(1-\tilde{\mu})pC_{\hbar}} - \alpha_4\frac{w_m-w_f}{\left(w_m+\tilde{\mu}w_f\right)(1+\tilde{\mu})}\right] <> 0$

If $\frac{d\tilde{\mu}}{dp} = 0$ then $\frac{dAverage\ Welfare\ \bar{w}}{dp} < 0$.

Proof of Proposition 4

Using (1'), (5a) and (2Ab):

$\frac{d\ Welfare\ \bar{s}}{dw_m} = \frac{\alpha_2}{w_m} + \frac{\alpha_3}{\left(w_m+pC_{\hbar}\right)\left(w_m+\tilde{\mu}w_f+(1-\tilde{\mu})pC_{\hbar}\right)}\left[\left(\tilde{\mu} - \frac{d\tilde{\mu}}{dw_m}\right)\left(w_f - pC_{\hbar}\right)\right] > 0$

Using (1'), (5a), (5b) and (2Ab):

$\frac{dWelfare\ \bar{w}}{dw_m} = \frac{\alpha_2}{w_m+w_f-pC_{\hbar}} - \frac{\alpha_3(w_f-pC_{\hbar})(1-\tilde{\mu})}{\left(w_m+pC_{\hbar}\right)\left(w_m+\tilde{\mu}w_f+(1-\tilde{\mu})pC_{\hbar}\right)} - \frac{\alpha_4(1+\tilde{\mu})}{\left(w_m+\tilde{\mu}w_f\right)(1+\tilde{\mu})} -$

$\frac{d\tilde{\mu}}{dw_m}\left[\frac{\alpha_3(w_f-pC_{\hbar})\left(w_m+w_f\right)}{\left(w_m+w_f\right)\left(w_m+\tilde{\mu}w_f+(1-\tilde{\mu})pC_{\hbar}\right)} - \frac{\alpha_4\left(w_m-w_f\right)}{\left(w_m+\tilde{\mu}w_f\right)(1+\tilde{\mu})}\right] <> 0$

Using Lemma 2, (5a), (5b) and (2Ab):

$$\frac{d\ Average\ Welfare\ \bar{w}}{dw_m} = \frac{\alpha_2}{w_m + w_f - pC_{\hbar}} - \frac{\alpha_3 (w_f - pC_{\hbar})(1-\tilde{\mu}\ )}{(w_m + pC_{\hbar})(w_m + \tilde{\mu} w_f + (1-\tilde{\mu}) pC_{\hbar})} - \frac{\alpha_4 (1+\tilde{\mu}\ )}{(w_m + \tilde{\mu}\ \ w_f)(1+\tilde{\mu}\ \ )} -$$

$$\frac{d\tilde{\mu}}{dw_m}\left[\frac{1}{2}\alpha_1 C_{\hbar} + \frac{\alpha_3 (w_f - pC_{\hbar})(w_m + w_f)}{(w_m + w_f)(w_m + \tilde{\mu} w_f + (1-\tilde{\mu}) pC_{\hbar})} - \frac{\alpha_4 (w_m - w_f)}{(w_m + \tilde{\mu}\ \ w_f)(1+\tilde{\mu}\ \ )}\right] <> 0$$

Proof of Proposition 5

Using (1'), (5a) and (2Ac):

$$\frac{d\ Welfare\ \bar{s}}{dw_f} = -\frac{\alpha_3\left[\tilde{\mu}\ \ + \frac{d\tilde{\mu}}{dw_f}(w_f - pC_{\hbar})\right]}{(w_m + pC_{\hbar})(w_m + \tilde{\mu} w_f + (1-\tilde{\mu}) pC_{\hbar})} < 0$$

Using (1'), (5a), (5b) and (2Ac):

$$\frac{dWelfare\ \bar{w}}{dw_f} = \frac{\alpha_2}{w_m + w_f - pC_{\hbar}} + \frac{\alpha_3 (w_m + pC_{\hbar})(1-\tilde{\mu}\ \ )}{(w_m + pC_{\hbar})(w_m + \tilde{\mu} w_f + (1-\tilde{\mu}) pC_{\hbar})} + \frac{\alpha_4 (w_m - \tilde{\mu}\ \ w_f)}{(w_m + \tilde{\mu}\ \ w_f) w_f} +$$

$$\frac{d\tilde{\mu}}{dw_f}\left[-\frac{\alpha_3 (w_f - pC_{\hbar})}{w_m + \tilde{\mu} w_f + (1-\tilde{\mu}) pC_{\hbar}} + \frac{\alpha_4 (w_m - w_f)}{(w_m + \tilde{\mu}\ \ w_f)(1+\tilde{\mu}\ \ )}\right] <> 0$$

If $\frac{d\tilde{\mu}}{dw_f} = 0$ then $\frac{dWelfare\ \bar{w}}{dw_f} > 0$.

Using Lemma 2:

$$\frac{d\ Average\ Welfare\ \bar{w}}{dw_f} = \frac{\alpha_2}{w_m + w_f - pC_{\hbar}} + \frac{\alpha_3 (w_m + pC_{\hbar})(1-\tilde{\mu}\ \ )}{(w_m + pC_{\hbar})(w_m + \tilde{\mu} w_f + (1-\tilde{\mu}) pC_{\hbar})} + \frac{\alpha_4 (w_m - \tilde{\mu}\ \ w_f)}{(w_{m,t} + \tilde{\mu}\ \ w_f) w_f} +$$

$$\frac{d\tilde{\mu}}{dw_f}\left[-\frac{1}{2}\alpha_1 C_{\hbar} - \frac{\alpha_3 (w_f - pC_{\hbar})}{w_m + \tilde{\mu} w_f + (1-\tilde{\mu}) pC_{\hbar}} + \frac{\alpha_4 (w_m - w_f)}{(w_m + \tilde{\mu}\ \ w_f)(1+\tilde{\mu}\ \ )}\right] <> 0$$

If $\frac{d\tilde{\mu}}{dw_f} = 0$ then $\frac{d\ Average\ Welfare\ \bar{w}}{dw_f} > 0$.

Stability condition for $\hat{\mu}_t$

Using (10) I derive the slope of $w_{f,t}$ as a function of $\tilde{\mu}_{t-1}$ as:

$$\frac{dw_{f,t}}{d\tilde{\mu}_{t-1}} = \beta(1-\theta)\tilde{\mu}_{t-1}^{\beta-1} w_m \geq 0 \quad \text{(4A.a)}$$

And thus

$$\frac{d^2 w_{f,t}}{d\tilde{\mu}_{t-1}^2} = \beta(\beta-1)(1-\theta)\tilde{\mu}_{t-1}^{\beta-2} w_m \leq (>) 0 \quad \text{iff } \beta \leq (>) 1 \quad \text{(4A.b)}$$

In steady state $\tilde{\mu}_{t-1} = \tilde{\mu}_t$

I rewrite (2Ac) to derive the slope:

$$\frac{dw_f}{d\tilde{\mu}} = \frac{\alpha_1 C_{\hbar} - \alpha_4 \frac{w_m - w_f}{(w_m + \tilde{\mu} w_f)(1+\tilde{\mu})}}{\left(\frac{\alpha_2}{w_m + w_f - pC_{\hbar}} + \frac{\alpha_3}{(w_m + w_f)} + \alpha_4 \frac{w_m}{w_f} \frac{1}{(w_m + \tilde{\mu} w_f)}\right)} \geq 0 \quad \text{(4A.c)}$$

and thus

$$\frac{d^2 w_f}{d\tilde{\mu}^2} = \frac{\alpha_4 \frac{(w_m - w_f)\left((w_m + w_f + 2\tilde{\mu} w_f)\right)}{\left[(w_m + \mu_t w_f)(1+\tilde{\mu})\right]^2}}{\left(\frac{\alpha_2}{w_m + w_f - pC_\hbar} + \frac{\alpha_3}{(w_m + w_f)} + \alpha_4 \frac{w_m}{w_f} \frac{1}{(w_m + \tilde{\mu} w_f)}\right)} + \frac{\left(\alpha_1 C_\hbar - \alpha_4 \frac{w_m - w_f}{(w_m + \tilde{\mu} w_f)(1+\tilde{\mu})}\right)\frac{\alpha_4 w_m}{(w_m + \tilde{\mu} w_f)^2}}{\left[\left(\frac{\alpha_2}{w_m + w_f - pC_\hbar} + \frac{\alpha_3}{(w_m + w_f)} + \alpha_4 \frac{w_m}{w_f} \frac{1}{(w_m + \tilde{\mu} w_f)}\right)\right]^2} > 0 \quad \text{(4A.d)}$$

If there exist an interior solution for $\hat{\mu}_t$ the stability condition for it is that

$$\beta(1-\theta)\tilde{\mu}_t^{\beta-1} w_m < \frac{\alpha_1 C_\hbar - \alpha_4 \frac{w_m - w_f}{(w_m + \tilde{\mu} w_f)(1+\tilde{\mu})}}{\left(\frac{\alpha_2}{w_m + w_f - pC_\hbar} + \frac{\alpha_3}{(w_m + w_f)} + \alpha_4 \frac{w_m}{w_f} \frac{1}{(w_m + \tilde{\mu} w_f)}\right)} \qquad \text{(4A.e)}$$

Proof of Proposition 6

Plug (10) into (6b) to determine $(\hat{\mu}, \widehat{w}_f)$ as well as the partial derivatives that reflect the effect that changes in the exogenous parameters w_m *and* p have on female labor force participation and female wages

$$Dd\hat{\mu} = dw_m \left[\alpha_2 \frac{pC_\hbar}{w_m\left(1+\theta+(1-\theta)\hat{\mu}^\beta\right) - pC_\hbar} + \alpha_3 \frac{pC_\hbar}{w_m(w_m + pC_\hbar)}\right] - dp \left[\alpha_2 \frac{C_\hbar}{w_m\left(1+\theta+(1-\theta)\hat{\mu}^\beta\right) - pC_\hbar} + \alpha_3 \frac{C_\hbar}{w_m + pC_\hbar}\right] \qquad \text{(5A.a)}$$

where

$$D = \alpha_1 C_\hbar - \alpha_2 \frac{w_m \beta(1-\theta)\hat{\mu}^{\beta-1}}{w_m\left(1+\theta+(1-\theta)\hat{\mu}^\beta\right) - pC_\hbar} - \alpha_3 \frac{\beta(1-\theta)\hat{\mu}^{\beta-1}}{1+\theta+(1-\theta)\hat{\mu}^\beta} - \alpha_4 \left[\frac{1-\theta-(1-\theta)\hat{\mu}^\beta}{\left(1+(\hat{\mu}\theta+(1-\theta)\hat{\mu}^\beta\right)(1+\hat{\mu})} + \frac{\beta(1-\theta)\hat{\mu}^{\beta-1}}{(\theta+(1-\theta)\hat{\mu}^\beta)\left(1+\hat{\mu}(\theta+(1-\theta)\hat{\mu}^\beta\right)}\right] \geq 0$$

Using (5Aa) and (4Ae) it is thus shown that if a stable interior solution exists

$$\frac{d\hat{\mu}_t}{dw_m} \geq 0 \qquad \text{(5A.b)}$$

and $\frac{d\hat{\mu}_t}{dp} \leq 0.$ (5A.c)

Using (10), (5Ab) and (5Ac):

$$\frac{d\widehat{w}_f}{dw_m} = \theta + (1-\theta)\hat{\mu}^\beta + \beta(1-\theta)\hat{\mu}^{\beta-1} \frac{d\hat{\mu}_t}{dw_m} > 0 \qquad \text{(6A.a)}$$

$$\frac{d\widehat{w}_f}{dp} = \beta(1-\theta)\hat{\mu}^{\beta-1} \frac{d\hat{\mu}}{dp} \leq 0 \qquad \text{(6A.b)}$$

$$\frac{d(\widehat{w}_f - w_m)}{dw_m} = \theta + (1-\theta)\hat{\mu}^\beta + \beta(1-\theta)\hat{\mu}^{\beta-1} \frac{d\hat{\mu}}{dw_m} - 1 >< 0 \qquad \text{(6A.c)}$$

Proof of Lemma 4

$$\frac{d\hat{\mu}}{d\alpha_3} = \frac{1}{D}\ln\left(\frac{w_m\left(1+\theta+(1-\theta)\hat{\mu}^{\beta}\right)}{w_m+pC_{\hbar}}\right) > 0$$

$$\frac{d\hat{\mu}}{d\alpha_4} = \frac{1}{D}\ln\left(\frac{\left(\theta+(1-\theta)\hat{\mu}^{\beta}\right)(1+\hat{\mu})}{\mathbf{1}+\hat{\mu}\ \ \left(\theta+(1-\theta)\hat{\mu}^{\beta}\right)}\right) < 0$$

$$\frac{d\widehat{w}_f}{d\alpha_3} = \ \beta(1-\theta)\hat{\mu}^{\ \beta-1}\frac{d\hat{\mu}}{d\alpha_3} > 0$$

$$\frac{d\widehat{w}_f}{d\alpha_4} = \ \beta(1-\theta)\hat{\mu}^{\ \beta-1}\frac{d\hat{\mu}}{d\alpha_4} < 0$$

Proof of Proposition 7

Using (3), (5Ac), (6Ab) and assumptions (5a) and (5b):

$$\frac{d\,\overline{Y^{\hbar\hbar}}}{dp} = \frac{d\hat{\mu}}{dp}\left(\widehat{w}_f - pC_{\hbar}\right) + \hat{\mu}\ \frac{d\widehat{w}_f}{dp} + (1-\hat{\mu})C_{\hbar} <> 0 \qquad \text{(7A.a)}$$

Using (4), (5Ac), (6Ab) and assumptions (5a) and (5b):

$$\frac{d\overline{Y^{ind}}}{dp} \ = \frac{\frac{d\hat{\mu}}{dp}\left(\widehat{w}_f - w_m\right)+(1-\hat{\mu})\hat{\mu}\,\frac{d\widehat{w}_f}{dp}}{(1+\hat{\mu})^2} <> 0 \qquad \text{(7A.b)}$$

Using (3), (5Ab), (6Aa) and assumptions (5a) and (5b):

$$\frac{d\,\overline{Y^{\hbar\hbar}}}{d\,w_m} = 1 + \frac{d\hat{\mu}}{dw_m}\left(\widehat{w}_f - pC_{\hbar}\right) + \hat{\mu}\ \frac{d\widehat{w}_f}{dw_m} > 0 \qquad \text{(7A.c)}$$

Using (4), (5Ab), (6Aa) and assumptions (5a) and (5b):

$$\frac{d\overline{Y^{ind}}}{d\,w_m} \ = \frac{\frac{d\hat{\mu}}{dw_m}\left(\widehat{w}_f - w_m\right)+\left(\frac{d\widehat{w}_f}{dw_m}+1\right)(1+\hat{\mu})}{(1+\hat{\mu})^2} > 0 \qquad \text{(7A.d)}$$

Proof of Proposition 8

Using (1'), (5a), (5Ac) and (6Ab):

$$\frac{d\ Welfare\ \bar{s}}{dp} = \frac{\alpha_3\left[\hat{\mu}\ C_{\hbar}\,\frac{w_m+\widehat{w}_f}{w_m+pC_{\hbar}} - \frac{d\hat{\mu}}{dp}\left(\widehat{w}_f - pC_{\hbar}\right) - \frac{d\widehat{w}_f}{dp}\hat{\mu}\ \right]}{w_m+\hat{\mu}\widehat{w}_f+(1-\hat{\mu})pC_{\hbar}} > 0$$

Using (1'), (5a), (5b), (5Ac) and (6Ab) it can be proven that:

$$\frac{dWelfare\,\bar{w}}{dp} = \frac{\alpha_2\left(\frac{d\hat{w}_f}{dp} - C_{\hbar}\right)}{w_m + \hat{w}_f - pC_{\hbar}} + \frac{\alpha_3\left[-(1-\hat{\mu})C_{\hbar} - \frac{d\hat{\mu}}{dp}(\hat{w}_f - pC_{\hbar}) + \frac{d\hat{w}_f}{dp}\frac{(1-\hat{\mu})(w_m + pC_{\hbar})}{w_m + \hat{w}_f}\right]}{w_m + \hat{\mu}\hat{w}_f + (1-\hat{\mu})pC_{\hbar}} +$$

$$\alpha_4 \frac{\frac{d\hat{\mu}}{dp}\hat{w}_f(w_m - \hat{w}_f) + \frac{d\hat{w}_f}{dp}w_m(1+\hat{\mu})}{\hat{w}_f(w_m + \hat{\mu}\hat{w}_f)(1+\hat{\mu})} <> 0$$

$$\frac{dAverage\ Welfare\,\bar{w}}{dp} = -\frac{d\hat{\mu}}{dp}\frac{1}{2}\alpha_1 C_{\hbar} + \frac{\alpha_2\left(\frac{d\hat{w}_f}{dp} - C_{\hbar}\right)}{w_m + \hat{w}_f - pC_{\hbar}} +$$

$$\frac{\alpha_3\left[-(1-\hat{\mu})C_{\hbar} - \frac{d\hat{\mu}}{dp}(\hat{w}_f - pC_{\hbar}) + \frac{d\hat{w}_f}{dp}\frac{(1-\hat{\mu})(w_m + pC_{\hbar})}{w_m + \hat{w}_f}\right]}{w_m + \hat{\mu}\hat{w}_f + (1-\hat{\mu})pC_{\hbar}} + \alpha_4 \frac{\frac{d\hat{\mu}}{dp}\hat{w}_f(w_m - \hat{w}_f) + \frac{d\hat{w}_f}{dp}w_m(1+\hat{\mu})}{\hat{w}_f(w_m + \hat{\mu}\hat{w}_f)(1+\hat{\mu})} <> 0$$

Proof of Proposition 9

Using (1'), (5a), (5Ab) and (6Aa):

$$\frac{d\ Welfare\,\bar{s}}{dw_m} = \frac{\alpha_2}{w_m} + \frac{\alpha_3\hat{\mu}\,(\hat{w}_f - pC_{\hbar})}{(w_m + pC_{\hbar})(w_m + \hat{\mu}\hat{w}_f + (1-\hat{\mu})pC_{\hbar})} - \frac{\alpha_3}{w_m + \hat{\mu}\hat{w}_f + (1-\hat{\mu})pC_{\hbar}}\left[\frac{d\hat{\mu}}{dw_m}(\hat{w}_f - pC_{\hbar}) + \hat{\mu}\,\frac{d\hat{w}_f}{dw_m}\right] <> 0$$

Using (1'), (5a), (5b), (5Ab) and (6Aa):

$$\frac{dWelfare\,\bar{w}}{dw_m} = \frac{\alpha_2\left(1 + \frac{d\hat{w}_f}{dw_m}\right)}{w_m + \hat{w}_f - pC_{\hbar}} + \frac{\alpha_3(1-\hat{\mu})\left[\frac{d\hat{w}_f}{dw_m}(w_m + pC_{\hbar}) - (\hat{w}_f - pC_{\hbar})\right]}{(w_m + \hat{w}_f)(w_m + \hat{\mu}\hat{w}_f + (1-\hat{\mu})pC_{\hbar})} +$$

$$\frac{\alpha_4\left[(1+\hat{\mu})\left(\frac{d\hat{w}_f}{dw_m}w_m - \hat{w}_f\right) + \frac{d\hat{\mu}}{dw_m}(w_m - \hat{w}_f)\right]\hat{w}_f}{\hat{w}_f(w_m + \hat{\mu}\hat{w}_f)(1+\hat{\mu})} <> 0$$

$$\frac{dAverageWelfare\,\bar{w}}{dw_m} = -\frac{d\hat{\mu}}{dw_m}\frac{1}{2}\alpha_1 C_{\hbar} + \frac{\alpha_2\left(1 + \frac{d\hat{w}_f}{dw_m}\right)}{w_m + \hat{w}_f - pC_{\hbar}} + \frac{\alpha_3(1-\hat{\mu})\left[\frac{d\hat{w}_f}{dw_m}(w_m + pC_{\hbar}) - (\hat{w}_f - pC_{\hbar})\right]}{(w_m + \hat{w}_f)(w_m + \hat{\mu}\hat{w}_f + (1-\hat{\mu})pC_{\hbar})} +$$

$$\frac{\alpha_4\left[(1+\hat{\mu})\left(\frac{d\hat{w}_f}{dw_m}w_m - \hat{w}_f\right) + \frac{d\hat{\mu}}{dw_m}(w_m - \hat{w}_f)\right]\hat{w}_f}{\hat{w}_f(w_m + \hat{\mu}\hat{w}_f)(1+\hat{\mu})} <> 0$$